1. FARADAY'S LAW & MAGNETIC FLUX

Magnetic Flux:

$$\Phi_B = B \cdot A \cdot \cos\theta = BA\cos\theta \quad \text{[Wb (Weber)]}$$

B = magnetic field strength (T), A = area (m^2), θ = angle between $\vec{B}$ and normal to surface

Faraday's Law of Electromagnetic Induction:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

Magnitude: $|\mathcal{E}| = \left|\frac{d\Phi_B}{dt}\right|$ [Volts] Negative sign indicates direction (Lenz's Law)

Induced EMF (Uniform Field Change):

$$\mathcal{E} = A\frac{\Delta B}{\Delta t}$$

For perpendicular uniform field change through a loop of fixed area.

Induced Current:

$$I = \frac{|\mathcal{E}|}{R} = \frac{1}{R}\left|\frac{d\Phi_B}{dt}\right| \quad \text{[Amperes]}$$

Common Pitfall

- ▷ EMF depends on **rate of flux change**, not flux itself
- ▷ Constant flux (even if large) → no induced EMF
- ▷ Changing flux (even if small) → EMF is induced
- ▷ Negative sign in Faraday's law gives **direction** via Lenz's law
- ▷ EMF = work per unit charge; not same as potential difference if resistance exists

2. LENZ'S LAW

Lenz's Law Statement: The induced current (or EMF) flows in a direction such that its magnetic field **opposes** the change in magnetic flux that produced it.

Practical Application:

- ▷ Flux increasing ⇒ induced current creates opposing field
- ▷ Flux decreasing ⇒ induced current tries to maintain field
- ▷ Direction found by: right-hand rule or Fleming's right-hand rule

Common Pitfall

- ▷ Lenz's law is a **consequence** of energy conservation, not an independent law
- ▷ Direction of induced current is determined by Lenz's law; magnitude by Faraday's law
- ▷ Induced magnetic field always opposes the **change**, not the original field

3. MOTIONAL EMF

Motional EMF (Straight Conductor Moving Perpendicular to B):

$$\mathcal{E} = Blv$$

B = magnetic field (T), l = length of conductor (m), v = velocity perpendicular to both $\vec{B}$ and conductor (m/s)

General Motional EMF (Vector Form):

$$\mathcal{E} = \int(\vec{v} \times \vec{B}) \cdot d\vec{l}$$

Force on Current-Carrying Conductor in Magnetic Field:

$$F = BIl \quad \text{[Newtons]}$$

Direction by Fleming's left-hand rule. Force always opposes motion (Lenz's law).

Terminal Velocity of Conductor in Magnetic Field: When applied force F_{app} equals magnetic force F_{mag}:

$$v_{terminal} = \frac{F_{app} \cdot R}{B^2 l^2}$$

Substituting $\mathcal{E} = Blv$ and $F_{mag} = BIl = \frac{B^2 l^2 v}{R}$

Power Dissipated:

$$P = \mathcal{E} \cdot I = \frac{(Blv)^2}{R} = F_{app} \cdot v_{terminal}$$

Common Pitfall

- ▷ EMF $= Blv$ works only when v is perpendicular to both $\vec{B}$ and the conductor
- ▷ Terminal velocity exists because induced EMF creates opposing force
- ▷ At terminal velocity: net force $= 0$, but acceleration $= 0$ (constant speed maintained)
- ▷ Power dissipated = Work done by applied force (energy conservation)

4. INDUCTANCE

Self-Inductance (L):

$$L = \frac{\Phi_B}{I} \quad \text{[Henry (H)]}$$

EMF due to self-inductance:

$$\mathcal{E}_L = -L\frac{dI}{dt}$$

Self-Inductance of Solenoid:

$$L = \mu_0 n^2 A l = \frac{\mu_0 N^2 A}{l}$$

n = turns per unit length, N = total turns, A = cross-sectional area (m^2), l = length (m) $\mu_0 = 4\pi \times 10^{-7}$ H/m

Self-Inductance of Toroid:

$$L = \frac{\mu_0 N^2 h}{2\pi} \ln\left(\frac{b}{a}\right)$$

h = height of toroid, a = inner radius, b = outer radius

Mutual Inductance (M):

$$M = k\sqrt{L_1 L_2}$$

k = coupling coefficient (0 k 1) EMF in secondary coil:

$$\mathcal{E}_2 = -M\frac{dI_1}{dt}$$

Common Pitfall

- ▷ $L \propto N^2$ (inductance is **quadratic** in number of turns)
- ▷ Coupling coefficient $k = 1$ only for perfectly wound coils (ideal case)
- ▷ Mutual inductance $M_{12} = M_{21}$ (reciprocal)
- ▷ Inductance opposes **change** in current, not current itself

5. RL CIRCUITS

Voltage Equation (RL Circuit):

$$V = IR + L\frac{dI}{dt}$$

Growing Current (Switch Closed at t = 0):

$$I(t) = \frac{V}{R}(1 - e^{-t/\tau}) = I_0(1 - e^{-t/\tau})$$

where $\tau = \frac{L}{R}$ is the time constant (seconds)
At $t = \tau$: $I = I_0(1 - e^{-1}) \approx 0.632 I_0$ (63.2% of final value)
Decaying Current (Switch Opened, Initial Current I_0):

$$I(t) = I_0 e^{-t/\tau} = I_0 e^{-Rt/L}$$

Energy Dissipated in Resistance:

$$E_R = \int_0^\infty I^2 R\, dt = \frac{1}{2} L I_0^2$$

All magnetic energy stored in inductor is dissipated as heat.

Common Pitfall

- ▷ Time constant $\tau = \frac{L}{R}$; larger $L \to$ slower rise, larger $R \to$ faster rise
- ▷ Current never truly reaches final value; it asymptotically approaches it
- ▷ L acts like inertia in electrical circuits; R acts like resistance/damping
- ▷ Voltage across inductor: $V_L = L\frac{dI}{dt}$ (maximum at $t = 0$)

6. AC CIRCUITS WITH INDUCTANCE (JEE FOCUS)

Inductive Reactance:

$$X_L = \omega L = 2\pi f L \quad \text{[Ohms]}$$

ω = angular frequency (rad/s), f = frequency (Hz)
X_L increases linearly with frequency (unlike capacitors).
Peak EMF and Peak Current (Pure Inductor):

$$I_0 = \frac{\mathcal{E}_0}{X_L} = \frac{\mathcal{E}_0}{\omega L}$$

Phase Relationship: In a pure inductor: **Current lags voltage by 90°**

$$V(t) = V_0 \sin(\omega t), \quad I(t) = I_0 \sin(\omega t - 90)$$

RMS Values:

$$I_{rms} = \frac{\mathcal{E}_{rms}}{X_L}, \quad P_{avg} = I_{rms}^2 \times 0 = 0$$

Reactive Power:

$$Q = I_{rms}^2 X_L = \frac{\mathcal{E}_{rms}^2}{X_L} \quad \text{[VAR]}$$

Common Pitfall

- ▷ Pure inductor: **no real power dissipation** ($P = 0$); all power is reactive
- ▷ $X_L \propto f$: doubling frequency → doubles reactance (unlike capacitors where $X_C \propto 1/f$)
- ▷ Phase lag = 90° for pure inductor; phase angle less for RL circuit with resistance
- ▷ Impedance of RL circuit: $Z = \sqrt{R^2 + X_L^2}$

7. TRANSFORMERS

Transformer Equation (Ideal Transformer):

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}$$

Subscripts: p = primary, s = secondary; N = number of turns
Power in Ideal Transformer:

$$P_p = P_s \quad \Rightarrow \quad V_p I_p = V_s I_s$$

No loss of power in ideal case.

Real Transformer (with losses):

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{V_s I_s \cos\phi_s}{V_p I_p \cos\phi_p} \times 100\%$$

Typical efficiency: 90-99% for real transformers.

EMF Relationship:

$$\frac{\mathcal{E}_s}{\mathcal{E}_p} = \frac{N_s}{N_p}$$

Step-Up Transformer: $N_s > N_p$ → voltage increases, current decreases **Step-Down Transformer:** $N_s < N_p$ → voltage decreases, current increases

Common Pitfall

- ▷ Transformers work **only with AC**; DC does not change flux → no induced EMF
- ▷ Ideal transformer assumption: no losses, 100% coupling ($k = 1$)
- ▷ Current and voltage are **inversely related**: higher voltage → lower current
- ▷ Transformer steps voltage, **not power**: $V_s I_s = V_p I_p$ (ideal)
- ▷ Core losses include hysteresis and eddy current losses

8. ENERGY AND POWER

Magnetic Energy Stored in Inductor:

$$U_L = \frac{1}{2} L I^2 \quad \text{[Joules]}$$

Energy density: $u = \frac{U_L}{V} = \frac{1}{2}\mu_0 B^2$ per unit volume

Energy Dissipated in Resistance:

$$E = I^2 R t \quad \text{[Joules]}$$

Instantaneous power: $P = I^2 R = \frac{V^2}{R}$ (Joule heating)

Work Done by External Agent (Moving Conductor):

$$W = \int F\,dx = \int B I l\,v\,dt = \int \mathcal{E} I\,dt$$

Power Balance in Moving Conductor:

$$P_{input} = P_{dissipated} + \frac{dU_L}{dt}$$

Common Pitfall

- ▷ Energy stored in inductor is **quadratic in current**: doubling I → 4× energy
- ▷ All stored magnetic energy dissipates as heat when circuit is suddenly opened
- ▷ Work done by external force = work against magnetic force + Joule heating

9. EDDY CURRENTS

Eddy Current Effects: Eddy currents are induced circular (whirlpool) currents in bulk conductors placed in **changing** magnetic fields.

Heat Generation:

$$Q = \int I_{eddy}^2 R\,dt$$

Applications:

- ▷ **Electromagnetic braking**: trains, elevators (energy → heat)
- ▷ **Damping**: eddy currents damp oscillations in moving coils

- ▷ **Metal detectors**: changing field induces eddy currents in metals

Reduction of Eddy Currents:

- ▷ Use laminated cores (thin sheets separated by insulation)
- ▷ Reduces eddy current paths → fewer losses

Common Pitfall

- ▷ Eddy currents require **changing** magnetic field; static field → no eddy currents
- ▷ Eddy current heating used in induction furnaces for melting metals
- ▷ Lamination reduces but does not eliminate eddy current losses

10. IMPORTANT RELATIONS

Ampere-Maxwell Law (Displacement Current):

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{d\vec{E}}{dt}$$

Symmetry between changing $\vec{E}$ producing $\vec{B}$ and changing $\vec{B}$ producing $\vec{E}$ (Faraday + Ampere-Maxwell).

Time Constant in RL Circuit:

$$\tau = \frac{L}{R} \quad \text{[seconds]}$$

Current reaches 63.2% of final value at $t = \tau$ (growth); decays to 36.8% after τ (decay).

Quality Factor (Q) for Resonance (RLC Circuit):

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Higher Q → sharper resonance peak, lower bandwidth, higher peak current at resonance.

Bandwidth:

$$\Delta f = \frac{f_0}{Q} = \frac{R}{\pi L}$$

Common Pitfall

- ▷ Quality factor Q is dimensionless; it measures ratio of energy stored to energy dissipated
- ▷ Higher resistance → lower Q (broader resonance); lower resistance → higher Q

11. QUICK REVISION TABLE – EMI FORMULAS

Concept	Formula	Units/Notes
Magnetic Flux	$\Phi_B = BA\cos\theta$	Wb (Weber)
Faraday's Law	$\mathcal{E} = -\frac{d\Phi_B}{dt}$	V (Volts)
Motional EMF	$\mathcal{E} = Blv$	V
Induced Current	$I = \frac{\mathcal{E}}{R}$	A (Amperes)
Self-Inductance (Solenoid)	$L = \mu_0 n^2 Al$	H (Henry)
Mutual Inductance	$M = k\sqrt{L_1 L_2}$	H
RL Growing Current	$I(t) = I_0(1 - e^{-t/\tau})$	$\tau = L/R$
RL Decaying Current	$I(t) = I_0 e^{-t/\tau}$	$\tau = L/R$
Inductive Reactance	$X_L = \omega L = 2\pi f L$	Ω (Ohms)
Transformer Ratio	$\frac{V_s}{V_p} = \frac{N_s}{N_p}$	Ideal transformer
Magnetic Energy	$U_L = \frac{1}{2}LI^2$	J (Joules)

12. MASTER: COMMON MISTAKES JEE/NEET

1. **EMF vs. Flux**: EMF depends on *rate of change* of flux, not flux itself. Constant flux $\rightarrow$ no EMF.
2. **Lenz's Law Direction**: Induced current opposes the *change* in flux, not the original flux direction.
3. **Motional EMF Direction**: Use **right-hand rule**: fingers in direction of $\vec{v}$, curl toward $\vec{B}$, thumb points in direction of force on positive charges.
4. **Terminal Velocity**: At terminal velocity, net force = 0, but continuous power is dissipated (not stored). External force work equals Joule heating.
5. **Self-Inductance Scaling**: $L \propto N^2$ (quadratic, not linear). Doubling turns $\rightarrow$ $4\times$ inductance.
6. **RL Time Constant**: $\tau = L/R$. Larger L or smaller R $\rightarrow$ longer time to reach steady state.
7. **AC Inductance**: Inductive reactance $X_L = 2\pi f L$ increases with frequency. At high frequencies, pure inductor acts almost like open circuit.
8. **Transformers & DC**: Transformers **cannot** step voltage for DC because DC does not produce changing magnetic flux.
9. **Power in Ideal Transformer**: $P_p = P_s$ (no loss). If $V_s > V_p$, then $I_s < I_p$ (inverse relationship).
10. **Eddy Currents**: Require *changing* magnetic field. Static field $\rightarrow$ no eddy currents. Lamination reduces losses.
11. **Energy in Inductor**: All magnetic energy $U_L = \frac{1}{2}LI^2$ dissipates instantly when circuit is opened (dangerous spike in voltage).
12. **Mutual Inductance**: $M_{12} = M_{21}$ (symmetric). Coupling coefficient $k \leq 1$ always.